Summer Work for A.P. Physics

Zachary High School (2010-2011)

Dr. Call

Instructions: There are several fields of physics that one can study: mechanics, electricity and magnetism, thermodynamics, etc. Next year, we will be focusing almost exclusively on mechanics, which is the study of how things move when they interact. A large portion of what we will learn centers around Newton's laws of motion. Our most important tool will be mathematics. We will also use various equipment for our labs, but because math is the best language we have to describe the motion of objects, a large component of the class will be math.

In particular, it will be important for you to feel comfortable manipulating algebraic expressions. I find that many students are not comfortable with algebraic expressions until *after* the numbers have been plugged in. But for a variety of reasons, which we will discuss in class, it is often desirable to *first* manipulate the equations, and wait till the very end to plug in the numbers. Consequently, a large portion of this packet focuses on manipulating algebraic expressions. Please complete the work in the packet for the first day of school; you may use scratch paper to work out your answers, but please write your final answers in the packet. Your completed packet is to be turned in on the first day of class and will be graded for completion. If you have questions that you cannot resolve on your own as you work through this material, you are welcome to email me at jay.call@zacharyschools.org for clarification or help. If you need help, don't be shy; I would love to hear from you.

(Throughout this packet: s = seconds, km = kilometers, m = meters, cm = centimeters, kg = kilograms.)

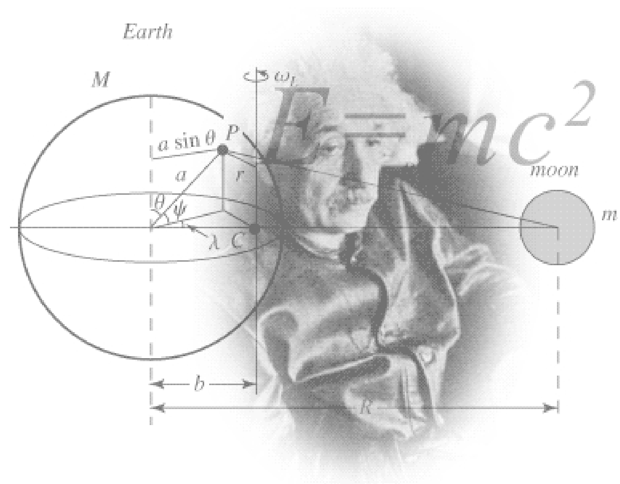


Figure taken from scienceworld.wolfram.com/physics.

Why Take AP Physics?

Haven't you heard? Physics is cool. It's true, we will be doing a lot of fun things in AP Physics next year. In addition to the book learning we will be doing, I have planned a number of exciting labs and demonstrations that will help you connect the principles we learn in the classroom with real life. Mastery of these principles will prepare AP students to excel on the AP exam next spring.

The central purpose of physics is to explore the way the world works. We want to understand *why* things act the way they do, especially when that behavior is counterintuitive. In our class, it will always be important to ask "why?". Once we understand why the world works the way it does, then our capacity to control the world is increased. (Think of light bulbs, airplanes, and computers. The list goes on...)

Interesting topics we will explore include:

1. free fall
2. vectors
3. projectiles
4. inertia and forces
5. Newton's laws of motion
6. friction
7. circular motion
8. terminal velocity
9. work, energy, and power
10. conservation laws
11. momentum and impulse
12. rocket propulsion
13. rotational motion and torque
14. gyroscopes
15. springs and pendula
16. law of gravity
17. planetary motion

Significant Digits: It should be no surprise to you that in science a number with units represents some physical quantity. But did you know that it also says something about the measurement process that produced it? For example, suppose Jill and Fred are dancing together at the Homecoming dance, and the distance between them is determined to be about 30 cm. It would be fair to say that the precision of the measured distance between them is on the order of tens of centimeters. In other words, no attempt was made to measure the distance between Jill and Fred down to the nearest micron. In fact, for reasons discussed below, it probably wouldn't make sense even to try.

So what would you think if someone told you that he had determined that the distance between Jill and Fred was actually 28.0026911014723 cm? That should set off red flags in your head. The inclusion of so many significant digits in this number implies that it is precise to within about 10^{-15} meters—the approximate size of a proton! I can immediately think of three big reasons why this is not likely to be the case.

1. It would be extremely difficult to measure distances on this scale without some very specialized tools. It's pretty hard to imagine someone trying to measure Jill and Fred with an eletron tunneling microscope or a laser interferometer out on the dance floor, but it sure is fun to try!

2. How does one determine endpoints for such a measurement? The thickness of a person can easily be 10 cm. Keeping this in mind, it really doesn't make sense to try and measure the distance between Jill and Fred to any accuracy better than tenths of meters. If you wanted to be really careful, you could define a person's position to be at their center of mass. If you had a way of determining where that was, you could start talking about more precise measurements of the distance between Jill and Fred.

3. Even with precise endpoints for your measurement, it still would not be practical to try and measure the distance between Jill and Fred to the nearest femtometer. Even the slightest breath of air on either of their parts would probably change the result of the measurement by a distance on the order of a millimeter. (That's a trillion femtometers!) And even if you could convince them to stop breathing, you'd still have their pulses to deal with. With Jill and Fred dancing, you can bet that the distance between their center of masses is always changing on a scale even bigger than that.

In our AP physics course, most of the numbers we use will have three significant digits. So, before you start the course, it is a good idea to make sure you know how to round numbers *correctly* to three significant digits. You probably think it's a snap, and in most cases it is, but there are two special cases that seem to confuse students.

1. For some reason, people tend to want to round a number like 6.999942 down. If we were rounding to five significant digits, then that would be the right thing to do (giving us 6.9999). But we want to round this number to three significant figures. The correct answer is 7.00, not 6.99. Make sure you understand why.

2. The second case involves what I like to call "hanging 5's". A hanging 5 shows up whenever the number you are trying to round is exactly half way between two values you might choose to round it to. For example, if you're rounding to three significant figures, then 69.35 has a hanging 5 (because it's exactly half way between 69.3 and 69.4), whereas 69.351 does not (because it's closer to 69.4). In elementary school you probably learned that you should always round 5's up. That's one simple approach to rounding, but it probably isn't the best from a statistical standpoint because it introduces a bias into your data. What does that mean? Essentially, it means that, on the average, you round up more than you round down. A better approach involves rounding hanging 5's up half of the time, and down the other half of the time. In AP physics, we will accomplish this by rounding hanging 5's such that the last significant digit of our rounded number is even. In other words, 69.35 would be rounded up to 69.4, but 69.25 would be rounded down to 69.2.

Round each of the following numbers to three significant digits. (The number of significant digits is determined by counting from left to right the number of digits, starting with the first *non-zero* digit. So 0.010, for example, has two significant digits, the one and the zero on the far right.)

1.	3.4177	______________	4.	12.99	______________	7.	0.1	______________
2.	0.73553	______________	5.	127.58	______________	8.	19.5	______________
3.	10,367	______________	6.	50.25	______________	9.	2.99999917	______________

Fractions: I understand that some of you are a little hesitant about taking a course that involves some calculus—especially if you've never had calculus. You may be surprised to know that, in my experience, students actually have more trouble with the algebra in AP physics than they do with the calculus! In fact, only a small amount of calculus will be required to succeed in this course, and I will gladly help you with any trouble you may have. Your algebra, geometry, and trig skills, on the other hand, should be well-polished. So the remaining material in the packet will all focus on those areas.

The first area I have noticed AP physics students struggle in deals with the manipulation of fractions. As practice, express the following quantities as a *single* fraction. No fractions should appear inside the numerator or denominator of your answer.

10. $\frac{c}{b} - \frac{c}{a}$ ________________

11. $\frac{A/B}{C/D} - 2$ ________________

12. $\frac{\left(k^{-1}r\right)/kr^{-1}}{k^2r^2}$ ________________

13. $\frac{3+x+x^2}{x} - 2$ ________________

14. $y + y^{-1}$ ________________

15. $\frac{x/y+1}{y/x+1}$ ________________

16. $\frac{t^3 + t\sin t}{\cos t} - \tan t$ ________________

17. $\frac{ax/(x+y)}{(x+y)/ay}$ ________________

18. $\frac{m/n^2}{m^2/n}$ ________________

19. $\frac{G}{M} - \frac{2G}{M}$ ________________

Exponents: I have also seen students struggle with exponents. Again, practice is really the best way to get your head around exponents. Make sure that you can easily express each of the following quantities as x to some power.

20. $\frac{1}{x}$ ________

21. $\left(x^3\right)^2$ ________

22. $x^a x^b$ ________

23. x ________

24. $\sqrt{x}$ ________

25. $\frac{1}{x^{-b}}$ ________

26. $\frac{x^5}{x^4}$ ________

27. 1 ________

28. $\frac{1}{\sqrt{x}}$ ________

29. $\left(x^a\right)^b$ ________

30. $\frac{x^2}{x^{-2}}$ ________

31. $\sqrt[3]{\sqrt{x}}$ ________

32. $\frac{1}{x^a}$ ________

33. $\sqrt{x^3}$ ________

34. $\frac{x^x}{x^2}$ ________

Functional Dependence: It's also very important that you're used to dealing with functions. The important thing to remember with functions is that mathematicians and scientists are typically more interested in how a particular function depends on its independent variable (the argument) than on what the function may be equal to in some particular case. In order to guarantee that you're on top of this, you should practice fiddling around with the way various functions depend on their arguments. The following problems will get you started.

Given that $f(x) = \frac{x^2}{1+x}$, $g(z) = z$, $h(t) = \frac{1}{t}$, and $p(x) = 72$, evaluate the following. You do *not* need to simplify.

32. $f(2)$ ________________

33. $f(x+y) - f(x)$ ________________

34. $f^2(x) - f\left(x^2\right)$ ________________

35. $f(0) - g(x)$ ________________

36. $f(\text{Fred}) - p(\clubsuit)$ ________________

37. $p(x+y)$ ________________

38. $p(0)$ ________________

39. $h\Big(f(r)\Big)$ ________________

Geometry: For this section, you just need to make sure you have these geometric formulas committed to memory or that you can use logic to quickly figure out what they are.

Write down a geometric expression for each of the following:

40. area of a circle with radius r ____________

41. circumference of a circle with radius r ____________

42. volume of a sphere with radius r ____________

43. surface area of a sphere with radius r ____________

44. volume of a cylinder with base radius r and height h ____________

45. surface area of a cylinder with base radius r and height h ____________

46. area of a triangle with base length b and height h ____________

47. area of a rectangle with width W and height h ____________

48. perimeter of a rectangle with width w and height h ____________

49. perimeter of a right triangle with horizontal leg b and vertical leg a ____________

50. area of a square with side length ℓ ____________

51. perimeter of a square with side length ℓ ____________

52. volume of a cube with edge length ℓ ____________

53. surface area of a cube with edge length ℓ ____________

54. area of a rhombus with base b and height h ____________

55. volume of a cone with base radius r and height h ____________

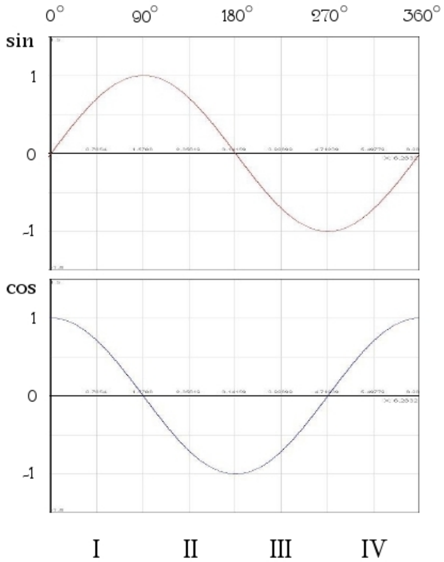
56. volume of a pyramid with base area A and height h ____________

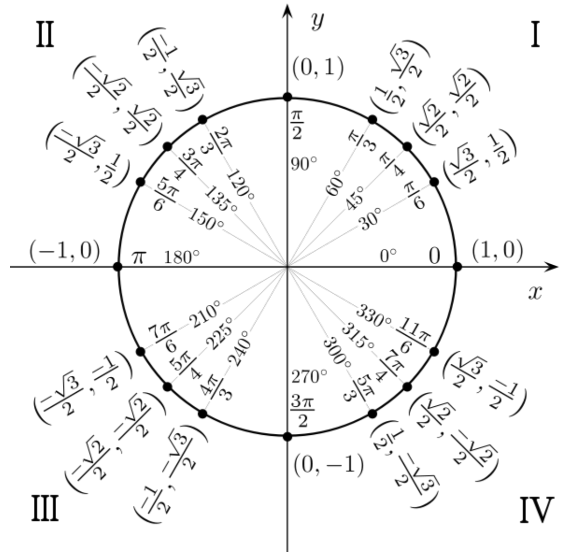
57. area of a trapezoid with base b, top a, and height h ____________

Trigonometry: You should know what the $\sin$ and $\cos$ functions evaluate to at each of the following first-quadrant angles: $0°, 30°, 45°, 60°$, and $90°$. In case you don't remember, it's actually pretty easy to learn these because of a neat pattern that emerges.

θ	$\sin\theta$		$\cos\theta$	
	pattern:	simplifies to:	pattern:	simplifies to:
$0°$	$\sqrt{0}/2$	0	$\sqrt{4}/2$	1
$30°$	$\sqrt{1}/2$	$1/2$	$\sqrt{3}/2$	$\sqrt{3}/2$
$45°$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2$
$60°$	$\sqrt{3}/2$	$\sqrt{3}/2$	$\sqrt{1}/2$	$1/2$
$90°$	$\sqrt{4}/2$	1	$\sqrt{0}/2$	0

You should also know what the $\sin$ and $\cos$ functions evaluate to at the analogous angles in each of the other three quadrants, namely: $120°, 135°, 150°, 180°, 210°, 225°, 240°, 270°, 300°, 315°, 330°$, and $360°$. The results in these three quadrants actually don't require any additional memorization because they're so similar to the values in the first quadrant (given in the table above). There are only two potential differences. The sign may be different, and the order that the values appear in the table may be reversed. So how can you be sure? I like to imagine little plots of the $\sin$ and $\cos$ functions in my head. That way, it's immediately obvious to me what the correct sign should be (positive whenever the function is above the x-axis and negative whenever its below the x-axis), and whether the values should be increasing or decreasing as θ increases (increasing if the slope is positive and decreasing if the slope is negative).





Another useful tool is the unit circle (a circle with a radius of 1 that happens to be centered at the origin). Since $\cos\theta$ is the x-component of the position of a point on the circle, the $\cos$ is negative in the second and third quadrants (on the left side of the unit circle, where x is less than zero), while the $\cos$ is positive in the first and fourth quadrants (on the right side of the unit circle, where x is greater than zero). Likewise, since $\sin\theta$ is the y-component of the position of a point on the circle, the $\sin$ is negative in the third and fourth quadrants (on the bottom half of the circle, where y is less than zero), while the $\sin$ is positive in the first and second quandrants (on the top half of the circle, where y is greater than zero). Similar arguments can help you determine where $\sin$ and $\cos$ are each increasing (with increasing θ) and where each of them is decreasing.

Then, knowing that $\tan\theta = \sin\theta/\cos\theta$, $\sec\theta = 1/\cos\theta$, and $\csc\theta = 1/\sin\theta$ for all values of θ, you should be able to quickly evaluate each of the six trig functions at each of these angles (without a calculator). As practice, evaluate the following trigonometric functions without looking at the chart or diagrams above.

58.	$\sin 45°$	________	60.	$\sin 30°$	________	62.	$\cos 30°$	________
59.	$\tan 60°$	________	61.	$\cos 240°$	________	63.	$\sec 0°$	________

True or False: Real problems frequently involve several mathematical and/or scientific principles at once. In practice, it can be easy to make little mistakes, even when you understand the associated principles fairly well. When this happens, it usually leads to a wrong answer. That's why it's always important to ask yourself if your answer makes sense. You should check both the numerical value of your answer, and the associated units. For example, of the following three statements, two are obviously wrong. Can you spot which ones?

1. The amount of time it takes to fly from Baton Rouge to Dallas is approximately 5.4×10^3 s.
2. The radius of Earth's orbit around the sun is approximately 3.19×10^2 m.
3. The speed of a bullet is approximately 1000 m/km.

Once you've determined that an answer is wrong, you should go back and check your work to try and determine where you made the first mistake (so that you can correct it!). Consequently, it's important for you to be able to identify little mistakes when you review your work. Hone your mistake-sniffing skills by determining which of the following statements are true ('T') and which are false ('F'). You should also try to identify what the mistake was that lead to any false statements. Good luck!

64. $\frac{a+b}{c} = \frac{a}{c} + \frac{b}{c}$ ___

65. $j - j\left(j^2 - j\right) = j - j^3 - j^2$ ___

66. $b^x b^{x+2} = b^{x(x+2)}$ ___

67. $e^{2x} - e^x = e^x$ ___

68. $(I-3)^2 = I^2 - 9$ ___

69. $(I-3)^2 = I^2 + 9$ ___

70. $x\log(x+1) = \log\Big(x(x+1)\Big)$ ___

71. $\log m - \log n = \log(m-n)$ ___

72. $\log a - \log b = \log \frac{a}{b}$ ___

73. $\sin a = -\sin(-a)$ ___

74. $\sin^2(t+1) + \cos^2(t+1) = 1$ ___

75. $\tan x = \frac{\sin x}{\cos x}$ ___

76. $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x \leq 0 \end{cases}$ ___

77. $\left(e^t\right)^{t+1} = e^{t(t+1)}$ ___

78. $\sin z = \sin(-z)$ ___

79. $\sin\theta = \cos\left(90^\circ - \theta\right)$ ___

80. $\cos g = \sin\left(g + 90^\circ\right)$ ___

81. $(x+y)(x-y) = x^2 - y^2$ ___

82. $(x+y)(x+y) = x^2 + y^2$ ___

83. $\frac{x}{y+z} = \frac{x}{y} + \frac{x}{z}$ ___

84. $H^{-1} = \frac{1}{H}$ ___

85. $2^3 2^5 = 2^{15}$ ___

86. $f^3 f^{-3} = 0$ ___

87. $s^{-2} = -\frac{1}{s^2}$ ___

88. $\left(x^2\right)^3 = x^5$ ___

89. $s^{-2} = \frac{1}{s^2}$ ___

90. $\frac{\ln a}{\ln b} = \log_b a$ ___

91. $\cos^2 t = \cos t^2$ ___

92. $m^{1/2} = \sqrt{m}$ ___

93. $\sin x = \frac{1}{\sec x}$ ___

94. $\cot x = \frac{\cos x}{\sin x}$ ___

95. $mn = nm$ ___

96. $\frac{xy}{z} = \frac{x}{z}\frac{y}{z}$ ___

97. $\frac{n^{m+3}}{n^5} = n^{m-2}$ ___

98. $\sin w = \cos w$ ___

99. $3^{-2} = \frac{1}{9}$ ___

100. $\frac{xy}{z} = \frac{x}{z}y$ ___

101. $\left(a^b\right)^c = \left(a^c\right)^b$ ___

102. $3(x+10) = 3x + 10$ ___

103. $y^k y^z = y^{k+z}$ ___

104. $p^3 p^{x+1} = p^{x+4}$ ___

105. $h^2 h^{-2} = 1$ ___

106. $\left(x^y\right)^z = x^{\left(y^z\right)}$ ___

107. $\left(r^2\right)^3 = r^6$ ___

108. $\sin^2 x = (\sin x)^2$ ___

109. $\log a + \log b = \log ab$ ___

110. $g\log(3h) = \log(3h)^g$ ___

111. $v^{1/3} = \sqrt[3]{v}$ ___

112. $t^{-1/2} = \frac{1}{\sqrt{t}}$ ___

113. $\sec\left(x^2\right) = \frac{1}{\cos\left(x^2\right)}$ ___

114. $(ab)c = a(bc)$ ___

115. $x\frac{y}{z} = \frac{x}{z}y$ ___

116. $\cos x = -\cos x$ ___

117. $\cos b = \cos(-b)$ ___

118. $16^{-1/2} = \frac{1}{4}$ ___

119. $(u+v)^3 = u^3 + 3u^2v + 3uv^2 + v^3$ ___